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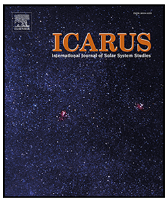

# Required deflection impulses as a function of time before impact for Earth-impacting asteroids

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ABSTRACT

For any asteroid on an impact trajectory, the amount of time prior to impact a deflection can be implemented can drastically change the amount of deflection impulse required. In this study we use the precision cloud-based asteroid orbit propagation and targeting capabilities of the Asteroid Institute's Asteroid Decision Analysis and Mapping (ADAM) platform to investigate the distribution of deflection $\Delta v$ required to divert asteroids on Earth-impacting trajectories (Chesley and Spahr, 2004) as a function of time prior to impact for 10,000 synthetic impacting asteroids. We target a miss distance of one Earth radius above the surface of the Earth and calculate the distribution of deflection $\Delta v$ required if applied 10, 20, 30, 40, and 50 years prior to impact. We find that the median required deflection impulse decreases as approximately $t^{-1}$ for increasing time before impact (Ahrens and Harris, 1992), where the median required $\Delta v$ is 1.4 cm/s, 0.76 cm/s, 0.55 cm/s, 0.46 cm/s, and 0.38 cm/s for 10, 20, 30, 40, & 50 years before impact, respectively. We find a considerable spread in the distribution of required deflection $\Delta v$ including a small fraction of asteroids that require an order of magnitude smaller or larger deflection $\Delta v$ than the median for each decade of time before impact studied.

## 1. Introduction

The Large Synoptic Survey Telescope (LSST; Ivezić et al., 2019) will increase the number of known near-Earth asteroids (NEAs) by more than an order of magnitude (Jones et al., 2018). Included in these ~100,000 newly discovered NEAs will be ~3,000 $H \leq 22$ potentially hazardous asteroids (PHAs) (LSST Science Collaborations and LSST Project, 2009; Eggl et al., 2019). Fortunately, asteroid impacts are natural disasters we have the ability to avoid. Here we consider the required magnitude of an impulsive change in velocity, or $\Delta v$, applied to an asteroid in order to prevent an impact with Earth. Understanding the $\Delta v$ requirements to deflect an asteroid on a collision course with Earth is critical to planetary defense. Impulsive $\Delta v$ can be applied by a number of deflection technologies including kinetic impactors and nuclear standoff explosions.

Demonstrations of the kinetic impactor method include the Deep Impact mission in 2005, colliding an impactor spacecraft with comet Tempel1 (Henderson and Blume, 2015), and the upcoming Double Asteroid Redirection Test (DART) mission, which will deflect the moon of the potentially hazardous asteroid (65803) Didymos in 2022 (Cheng et al., 2018).

Most of the previous studies of asteroid deflection using the kinetic impactor method have focused on mission concepts and planning requirements for a potential real-life impact deflection scenario. These types of studies depend on the specific properties of the asteroid, such as composition, mass, and the rotational and translational state of the asteroid (Bruck Syal et al., 2016; Zhang et al., 2017; Graninger et al., 2018; Remington et al., 2018) for determining the spacecraft requirements for successful deflection. The end goal of such studies is to determine the resulting $\Delta v$ given a set of asteroid structure and composition assumptions.

In our study, we focus solely on the orbital dynamics of asteroid deflection and the required deflection $\Delta v$ as a function of time before impact for a large sample of impacting asteroids. The modified orbit of potential Earth impactors is a function of the affected $\Delta v$ and does not depend on the composition or other physical properties of the asteroid. This is true for any deflection method, including a kinetic impactor or nuclear standoff explosion. The distribution of required $\Delta v$ is needed to understand the range of asteroid deflection scenarios. And, as described below, the $\Delta v$ required for deflection changes as a function of time, and therefore the $\Delta v$ evolution over time is also needed for mission planning and launch window calculations.

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Ahrens and Harris (1992) provided an order-of-magnitude analytical estimate of the required deflection $\Delta v$ for the circular two-body problem that found the $\Delta v$ is inversely proportional to the time before impact. (See Section 4 for a direct comparison between the results presented in this paper and the approximation made by Ahrens and Harris (1992).) This rough analytical estimate was shown to be in agreement with the results from Carusi et al. (2002) who numerically integrated a small sample of synthetic near-Earth objects (NEOs) modified from NEOs in the MPC catalog to calculate the required along-track deflection $\Delta v$ as a function of time prior to impact for their handful of objects. They then compared their results to the analytical estimates of Ahrens and Harris (1992) and Valsecchi et al. (2003).

The goal of this study is to determine the distribution of required deflection impulses as a function of time before impact for 10,000 synthetic impacting asteroids while also providing insight into possible atypical impact scenarios. We utilize the precision cloud-based asteroid orbit propagation and targeting capabilities of the Asteroid Institute's Asteroid Decision Analysis and Mapping (ADAM) platform to perform our calculation.

## 2. Asteroid decision analysis and mapping (ADAM) platform

The study of asteroids and asteroid populations requires manipulation of observational data, simulations, and analysis. The majority of the tools and data sets created by the scientific community have been built to run on individual workstations, making large-scale studies time-consuming. In addition, it is often difficult to configure these tools to match the many different coordinate frames and time standards commonly used by the scientific community, making comparisons cumbersome.

The vision of ADAM is to create a cloud-based platform available to the scientific community that provides a unified interface to multiple tools and enables large-scale studies. ADAM will include pre-configured settings to match common practices, such as the use of various time standards and coordinate frames, removing the need for the user to perform any necessary transformations for comparison to results from external tools. ADAM's architecture consists of a web-service front-end, cloud-based storage, and cloud-based compute engines encapsulating multiple tools for computation and analysis.

ADAM implements tools for orbit propagation, orbit determination (in progress), and analysis, such as telescope observation windows. Eventually, ADAM will include the additional benefits of visualizing computed orbits and trajectories. The cloud-based implementation of ADAM provides fast orbit propagation of large-scale populations of asteroids due to its ability to parallelize computations on a large number of compute cores. ADAM's current orbit propagation capabilities utilize Analytical Graphics Inc. (AGI)'s Systems Tool Kit (STK) Components Segmented Propagation Library, which is the first library made available via ADAM. Since its release as part of STK in 1998, the STK astrodynamics software "Astrogator" (included in Components) has been used internationally for pre-launch mission analysis and in operations on numerous commercial and government Earth-orbiting, lunar, and deep space missions, such as the WMAP, CONTOUR, DISCOVR, LCROSS, IBEX, LADEE, LRO, MAVEN, MESSENGER, and New Horizons missions. The required deflection $\Delta v$ distributions provided in this study were computed using ADAM's targeted propagation capabilities, which utilize the Astrogator software.

## 3. Methods

In this study we use the population of 10,000 synthetic impactors described by Chesley and Spahr (2004). While there are some differences between this population and the observed distribution of bolide $v_\infty$ (S. Chesley personal communication, 2016), this distribution serves as a reasonable starting point for our purposes of studying the distribution of impactor deflection $\Delta v$. To update the impactor population from using the older DE405 force model to the newer DE430 force model, the impactor velocities were corrected under the newer model to match the original impact locations.

To calculate the $\Delta v$ needed to move a trajectory from hitting the Earth to a designated miss distance, we implemented a shooting-method "targeter" in ADAM, which is a differential corrector numerical technique. The differential corrector uses a Newton–Raphson algorithm to achieve (or "target") a specified miss distance constraint by varying the velocity ("along-track") component of the $\Delta v$ vector. For this study, we only considered deflection $\Delta v$ along the velocity direction, either in the prograde or retrograde direction, since it is well known that when applied many orbits prior to impact, these along-track deflections are more effective than out-of-plane or radial deflections and are cumulative with each orbit, whereas the effect of out-of-plane and radial deflections is oscillatory. This restriction can be relaxed in a future study, but we expect it to have little effect on our conclusions.

The implementation of the targeter in STK Components calculates partial derivatives numerically by running a nominal trajectory and then trajectories with a slightly perturbed maneuver. The resulting sensitivity matrix of partial numerical derivatives is a linear approximation of how the targeted constraint values vary when the control parameters are altered. The differential corrector iterates, calculating new partial derivatives on each iteration, to account for any non-linearity in the trajectory problem. The sensitivity matrix is inverted using a singular value decomposition numerical method.

The targeting problem is implemented in two stages. In the first stage, the targeter varies the along-track component of $\Delta v$ to target the B-Plane vector magnitude to be 10 times farther than the desired miss distance. This "B Vector" is the perpendicular distance from the center of the Earth to the incoming trajectory asymptote (Kizner, 1961; Opik, 1976; Greenberg et al., 1988). We empirically find that coarsely targeting on this distance provides a good first guess for the next stage. Once the first stage converges, the second stage varies the along-track $\Delta v$ to finely target the desired miss distance measured at closest approach. This two-step process is unnecessary for most deflections, but for those that have a velocity vector nearly parallel to the Earth's (see Section 5.2) with a slow relative velocity, this technique was needed for convergence.

We set the desired miss distance to be one Earth radius (i.e., two Earth radii from the center of the Earth). This distance was chosen as a reasonable compromise between maximizing the miss distance from Earth while minimizing the deflection difficulty.

In general, the deflection $\Delta v$ will vary throughout an individual orbit with deflections near perihelion (closest orbital distance to the Sun) being more effective and deflections at aphelion (farthest orbital distance from the Sun) being relatively less effective. Fig. 1 shows the "ringing" present in the required deflection $\Delta v$ over a 10-year period for an example impacting asteroid to miss the Earth by 2 Earth radii from Earth's center. This ringing effect was also found in Carusi et al. (2002) (see Section 1). This effect is caused by the fact that for a given $\Delta v$, the greater orbital speed (and thus orbital energy) at perihelion results in a larger change in orbital kinetic energy, resulting in a larger change in orbital period. In addition, orbital inclination also affects required deflection impulses. When a deflection is applied to a highly inclined object while it is far from the ecliptic, a higher $\Delta v$ is required than a deflection impulse applied when the object is at a point along its orbit that is close to the ecliptic.

Rather than choosing the ideal location along an asteroid's orbit to apply the deflection $\Delta v$, we chose to apply the $\Delta v$ at set times prior to impact in order to get a representative spread in the deflection requirements for a large sample of impacting asteroids at all points along their orbits regardless of our ability or the difficulty to deflect the asteroids at those orbital locations. In a real-life impact deflection scenario, we may not have the chance to choose to deflect the asteroid at the ideal location along its orbit due to a limited amount of time before impact for launch of a spacecraft and rendezvous with the

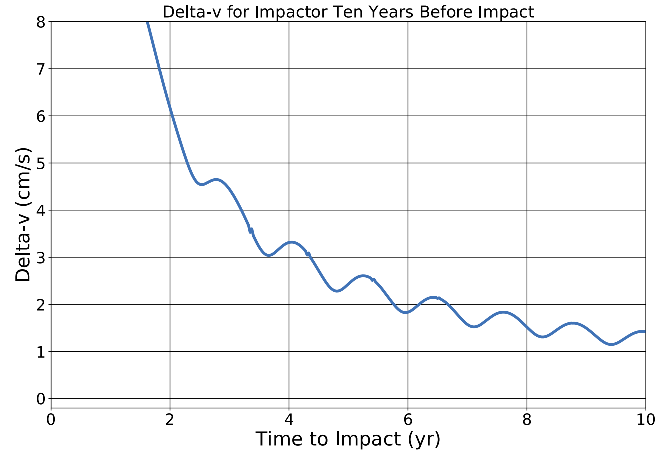

**Fig. 1.** Required $\Delta v$ to deflect an asteroid by a miss distance of 2 Earth radii from Earth's center as a function of time before impact. The oscillations represent the varying deflection requirements along an asteroid's orbit as it travels between perihelion (its closest distance to the Sun) and aphelion (its farthest distance from the Sun).

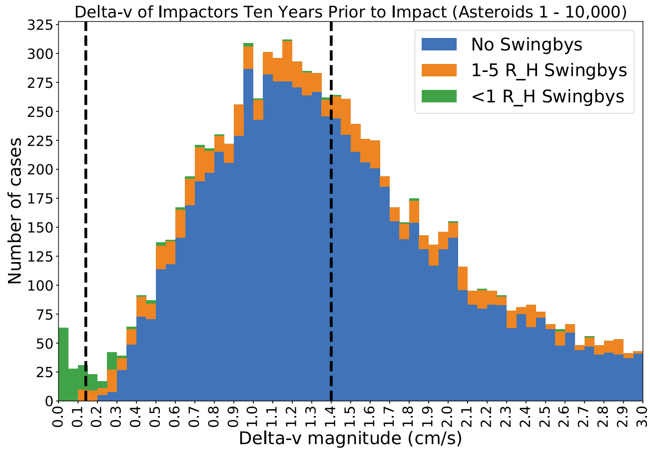

**Fig. 2.** Stacked histogram of required deflection $\Delta v$ distribution for 10,000 synthetic Earth impactors to miss Earth by one Earth radius (i.e., two Earth radii from Earth's center) at 10 years prior to impact. Impactors that experience planetary close approaches within 1 $R_H$ of a planet in the 10 years leading up to impact are shown in green, those with planetary close encounters within 1–5 $R_H$ of a planet are shown in orange, and those that do not experience any close encounters within 5 $R_H$ of a planet are shown in blue. The vertical lines at 1.4 cm/s and 0.14 cm/s mark the median and order-of-magnitude-below-the-median $\Delta v$ values, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

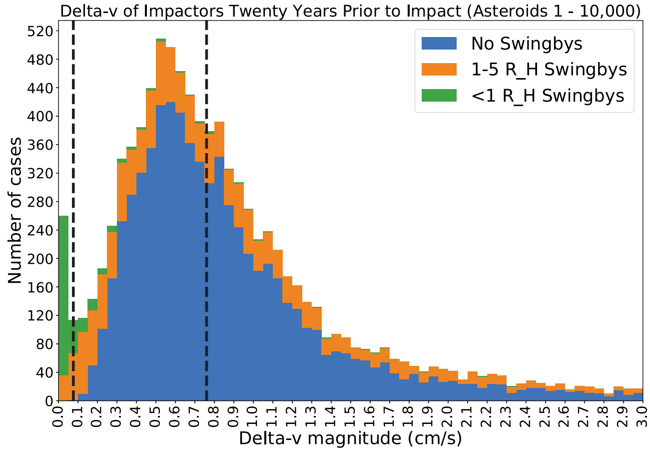

**Fig. 3.** Stacked histogram of required deflection $\Delta v$ distribution for 10,000 synthetic Earth impactors to miss Earth by one Earth radius (i.e., two Earth radii from Earth's center) at 20 years prior to impact. Impactors that experience planetary close approaches within 1 $R_H$ of a planet in the 20 years leading up to impact are shown in green, those with planetary close encounters within 1–5 $R_H$ of a planet are shown in orange, and those that do not experience any close encounters within 5 $R_H$ of a planet are shown in blue. The vertical lines at 0.76 cm/s and 0.076 cm/s mark the median and order-of-magnitude-below-the-median $\Delta v$ values, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

asteroid, so our study focuses on finding a representative sample of required deflection $\Delta v$ for all locations along an asteroid's orbit.

As discussed in Section 1, our study also does not focus on the effect of asteroid composition or other properties on asteroid deflection, but rather the orbital dynamics of changing a potential impactor's orbital period, which depends solely on the orbit of the asteroid.

## 4. Deflection impulses as a function of time before impact

Using ADAM's differential corrector (i.e., targeter), we compute the deflection $\Delta v$ distribution required for the 10,000 synthetic impacting asteroids described in Section 3 to miss Earth by one Earth radius (i.e., two Earth radii from the center of the Earth) when the $\Delta v$ is applied 10, 20, 30, 40, and 50 years before impact. Figs. 2–6 show stacked histograms of the required deflection $\Delta v$ when applied 10–50 years before impact, respectively. The required $\Delta v$ for impactors that experience planetary close encounters both within 5 Hill sphere radii $R_H$ of a planet and within 1 $R_H$ are shown separately from those that do not. (Sections 5.1 and 5.2 provide more detailed discussion on the effect of planetary close encounters on the distribution of required deflection $\Delta v$.) The impacting asteroids we are most likely to encounter would require a deflection $\Delta v$ between a few tenths of a cm/s and several cm/s, depending on the time available before impact to change the asteroid's speed.

Table 1 contains the $\Delta v$ distribution peak, mean, and median required at 10, 20, 30, 40, and 50 years before impact. We find that the median required $\Delta v$ decreases as $\approx t^{-1}$ for increasing time before impact. For example, the median required $\Delta v$ at 10 years prior to impact (1.4 cm/s) is roughly twice that required at 20 years before impact (0.76 cm/s).

This is in agreement with the order-of-magnitude analytical estimate from Ahrens and Harris (1992) of the required deflection $\Delta v$ for the circular two-body approximation when the deflection $\Delta v$ is applied parallel to the orbital motion of the asteroid. The relationship between the deflection $\Delta v$ and the time before impact in Ahrens and Harris (1992) for such a scenario is found to be $\Delta v \approx R_\oplus/3t \approx 0.07\ \mathrm{m\ s^{-1}/t_{years}}$ for a 1 $R_\oplus$ deflection (their Eqn. 7). For a 2 $R_\oplus$ deflection, as we use in our study, this becomes $\Delta v \approx 2R_\oplus/3t \approx 0.13\ \mathrm{m\ s^{-1}/t_{years}}$. For a deflection that occurs 10 years before impact with a miss distance 2 Earth radii from the center of the Earth, for example, this equation results in a required deflection $\Delta v$ of 1.3 cm/s, which is consistent with our median required $\Delta v$ at 10 years prior to impact for this miss distance of 1.4 cm/s.

In the case of an eccentric orbit, the orbital period modification can be magnified when the deflection impulse is applied at perihelion, thus requiring less $\Delta v$ to avoid a collision. For an eccentricity of 0.5, $\Delta v \approx R_\oplus/5t \approx 0.04\ \mathrm{m\ s^{-1}/t_{years}}$ for a 1 $R_\oplus$ deflection (Ahrens and Harris, 1992). Although the medians of our deflection $\Delta v$ distributions are consistent with the circular two-body approximation from Ahrens and Harris (1992), which is not surprising, it is clear from Figs. 2–6 that a wide range of deflection impulses are required for our synthetic impactor population. This is due to several factors, including the broad range of orbits (and eccentricities) of the impacting asteroids, the variety of points along the asteroid orbits at which the deflections occur, and the impact geometry at the point of intersection between

**Table 1**
Deflection $\Delta v$ distribution peak, mean, and median required for deflecting 10,000 synthetic impacting asteroids by one Earth radius (i.e., two Earth radii from Earth's center) at 10, 20, 30, 40, and 50 years prior to impact. The last two columns contain the percentage of impactors that experience planetary close encounters (PCEs).

| Time before impact (yr) | Peak $\Delta v$ (cm/s) | Mean $\Delta v$ (cm/s) | Median $\Delta v$ (cm/s) | $<1R_H$ planetary close encounters (%) | $<5R_H$ planetary close encounters (%) |
|---|---|---|---|---|---|
| 10 | 1.2 | 2.3 | 1.4 | 2.1 | 14.8 |
| 20 | 0.55 | 2.3 | 0.76 | 3.9 | 28.3 |
| 30 | 0.45 | 2.6 | 0.55 | 6.1 | 39.4 |
| 40 | 0.30 | 3.0 | 0.46 | 8.4 | 59.8 |
| 50 | ~0.2 | 2.9 | 0.38 | 11.7 | 60.0 |

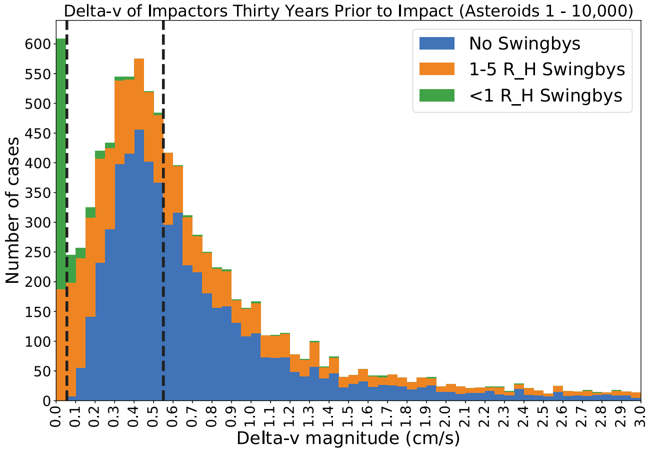

**Fig. 4.** Stacked histogram of required deflection $\Delta v$ distribution for 10,000 synthetic Earth impactors to miss Earth by one Earth radius (i.e., two Earth radii from Earth's center) at 30 years prior to impact. Impactors that experience planetary close approaches within 1 $R_H$ of a planet in the 30 years leading up to impact are shown in green, those with planetary close encounters within 1–5 $R_H$ of a planet are shown in orange, and those that do not experience any close encounters within 5 $R_H$ of a planet are shown in blue. The vertical lines at 0.55 cm/s and 0.055 cm/s mark the median and order-of-magnitude-below-the-median $\Delta v$ values, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

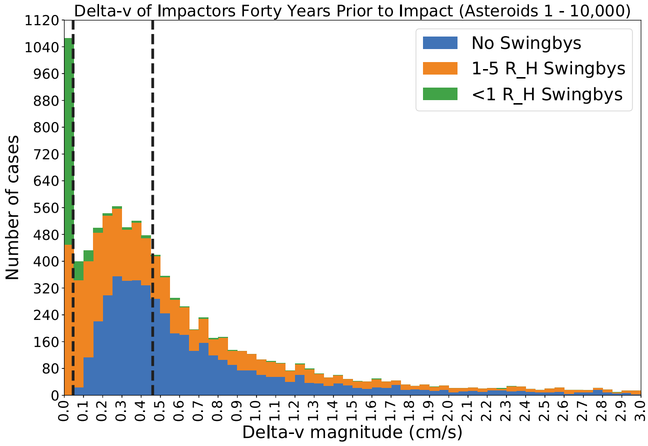

**Fig. 5.** Stacked histogram of required deflection $\Delta v$ distribution for 10,000 synthetic Earth impactors to miss Earth by one Earth radius (i.e., two Earth radii from Earth's center) at 40 years prior to impact. Impactors that experience planetary close approaches within 1 $R_H$ of a planet in the 40 years leading up to impact are shown in green, those with planetary close encounters within 1–5 $R_H$ of a planet are shown in orange, and those that do not experience any close encounters within 5 $R_H$ of a planet are shown in blue. The vertical lines at 0.46 cm/s and 0.046 cm/s mark the median and order-of-magnitude-below-the-median $\Delta v$ values, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

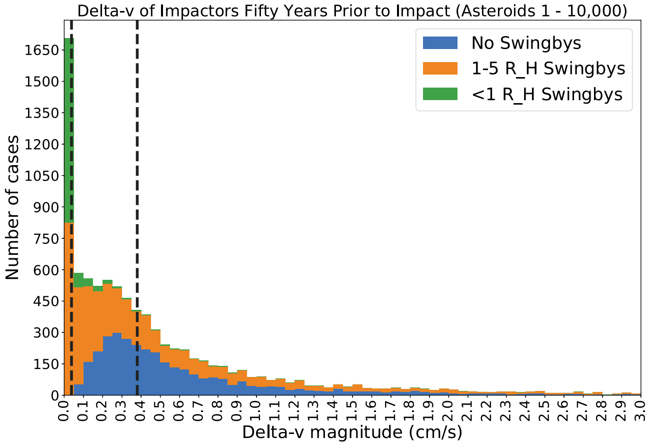

**Fig. 6.** Stacked histogram of required deflection $\Delta v$ distribution for 10,000 synthetic Earth impactors to miss Earth by one Earth radius (i.e., two Earth radii from Earth's center) at 50 years prior to impact. Impactors that experience planetary close approaches within 1 $R_H$ of a planet in the 50 years leading up to impact are shown in green, those with planetary close encounters within 1–5 $R_H$ of a planet are shown in orange, and those that do not experience any close encounters within 5 $R_H$ of a planet are shown in blue. The vertical lines at 0.38 cm/s and 0.038 cm/s mark the median and order-of-magnitude-below-the-median $\Delta v$ values, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the orbits of the asteroid and Earth (see Sections 5.1 and 5.2 for more discussion).

As shown in Figs. 2–6, the lowest $\Delta v$ bin ($\Delta v \leq 0.05$ cm/s) holds an increasing fraction of impactors with increasing time before impact. Quantitatively, we find that 1.2%, 3.3%, 6.4%, 10.1%, and 15.0% of impacting asteroids for 10, 20, 30, 40, and 50 years before impact, respectively, require $>10$ times less velocity impulse than the median $\Delta v$ for each decade (i.e., $\Delta v \leq 0.14$ cm/s, 0.076 cm/s, 0.055 cm/s, 0.046 cm/s, and 0.038 cm/s, respectively, for 10, 20, 30, 40, and 50 years prior to impact; see Table 2).

We also find an increasing number of impactors that result in required $\Delta v$ that fall below the threshold of ADAM's targeting capabilities (i.e., $\Delta v < 1 \times 10^{-7}$ cm/s). These impactors constitute 0.06%, 0.34%, 1.2%, 3.0%, and 6.3% of the impacting population for 10, 20, 30, 40, and 50 years prior to impact, respectively. The calculated $\Delta v$ for these impactors is small enough that numerical noise in the orbit propagations alone can cause them to miss the Earth by the specified miss distance. These very low-$\Delta v$ impact scenarios are discussed in more detail in Section 5.1.

On the other end of the spectrum, we find a long tail in the required deflection $\Delta v$ distribution for all five decades studied before impact. These tails stretch out to $\Delta v$ equal to tens of m/s, although we find that only 1.0%, 2.9%, 4.7%, 6.1%, and 7.8% of impactors for 10, 20, 30, 40, and 50 years prior to impact, respectively, require an order of magnitude higher $\Delta v$ than the median required $\Delta v$ (i.e., $\Delta v \geq 14$ cm/s, 7.6 cm/s, 5.5 cm/s, 4.6 cm/s, and 3.8 cm/s, respectively, for 10, 20, 30, 40, and 50 years prior to impact; see Table 3). A description of these high-$\Delta v$ impactors can be found in Section 5.2.

**Table 2**

Column 2: Percentage of the impacting population that require an order of magnitude less $\Delta v$ than the median required $\Delta v$ for each decade of time prior to impact studied. Column 3: Percentage of those impactors with a required deflection $\Delta v$ an order of magnitude below the median $\Delta v$ that experience planetary close encounters (PCEs) within 1 $R_H$ of a planet. Column 4: Percentage of those impactors that undergo PCEs within 1 $R_H$ of a planet that have a required deflection $\Delta v$ an order of magnitude below the median $\Delta v$. Column 5: Percentage of those impactors with a required deflection $\Delta v$ an order of magnitude below the median $\Delta v$ that experience PCEs within 5 $R_H$ of a planet. Column 6: Percentage of those impactors that undergo PCEs within 5 $R_H$ of a planet that have a required deflection $\Delta v$ an order of magnitude below the median $\Delta v$.

| Time before impact (yr) | $\Delta v \leq (\Delta v_{median})/10$ (%) | % of $\Delta v \leq (\Delta v_{median})/10$ with $< 1R_H$ PCEs | % of $< 1R_H$ PCEs with $\Delta v \leq (\Delta v_{median})/10$ | % of $\Delta v \leq (\Delta v_{median})/10$ with $< 5R_H$ PCEs | % of $< 5R_H$ PCEs with $\Delta v \leq (\Delta v_{median})/10$ |
|---|---|---|---|---|---|
| 10 | 1.2 | 94 | 52 | 100 | 7.9 |
| 20 | 3.3 | 77 | 66 | 99.7 | 10.5 |
| 30 | 6.4 | 64 | 73 | 99.7 | 13.5 |
| 40 | 10.1 | 60 | 78 | 99.9 | 14.1 |
| 50 | 15.0 | 56 | 78 | 100 | 14.6 |

**Table 3**

Column 2: Percentage of the impacting population that require an order of magnitude more $\Delta v$ than the median required $\Delta v$ for each decade. Column 3: Percentage of impactors that require more than an order of magnitude more $\Delta v$ than the median (i.e., a high-$\Delta v$) for which the asteroid's heliocentric velocity at infinity is nearly parallel (within 15°) to that of the Earth. Column 4: Those impactors that undergo planetary close encounters (PCEs) within 5 $R_H$ of a planet that have a required deflection $\Delta v$ an order of magnitude above the median $\Delta v$. Column 5: Those impactors with a required deflection $\Delta v$ an order of magnitude above the median $\Delta v$ that experience PCEs within 5 $R_H$ of a planet.

| Time before impact (yr) | $\Delta v \geq (\Delta v_{median})*10$ (%) | % of high-$\Delta v$ with parallel orbits | % of $< 5R_H$ PCEs with $\Delta v \geq (\Delta v_{median})*10$ | % of $\Delta v \geq (\Delta v_{median})*10$ with $< 5R_H$ PCEs |
|---|---|---|---|---|
| 10 | 1.0 | 65 | 69 | 81 |
| 20 | 2.9 | 66 | 61 | 68 |
| 30 | 4.7 | 63 | 44 | 59 |
| 40 | 6.1 | 59 | 27 | 47 |
| 50 | 7.8 | 54 | 27 | 43 |

## 5. Examples of interesting Earth impact scenarios

In this section, we describe examples of interesting Earth impact scenarios that require deflection $\Delta v$ far from the median value. This includes the low-$\Delta v$ and high-$\Delta v$ deflection requirements briefly described at the end of Section 4.

### 5.1. Earth impactors that require little deflection $\Delta v$

As described in Section 4, we find an increasing fraction of impactors that require an order of magnitude less $\Delta v$ than the median $\Delta v$ (i.e., $\Delta v \leq 0.14$ cm/s, 0.076 cm/s, 0.055 cm/s, 0.046 cm/s, and 0.038 cm/s, respectively, for 10, 20, 30, 40, and 50 years prior to impact) for increasing time before impact. This can be seen in Figs. 2–6. In particular, we find that 1.2%, 3.3%, 6.4%, 10.1%, and 15.0% of impactors have required $\Delta v$ <10 times the median $\Delta v$ for 10, 20, 30, 40, and 50 years prior to impact, respectively (Table 2).

A majority (94%, 77%, 64%, 60%, and 56% for 10, 20, 30, 40, & 50 years before impact, respectively) of these impactors experience planetary close encounters (within one Hill sphere radius $R_H$ of a planet) prior to impact (Table 2). We find that 95% of the time these close encounters occur with the Earth itself. Although this fraction decreases with time before impact, when the close approach distance is relaxed to be within 5 $R_H$ of a planet, we find that virtually all impactors that required $\Delta v$ <10 times the median $\Delta v$ experience close encounters within this distance; roughly 66% of these close approaches occur with Earth. Thus, more distant close approaches can also alter the required deflection $\Delta v$ and explains the drop seen in the fractions above.

Overall, a fraction (14.8%, 28.3%, 39.4%, 59.8%, and 60.0% for 10, 20, 30, 40, and 50 years prior to impact, respectively) of all impactors experience planetary close encounters within 5 $R_H$ of a planet during the time before impact (Table 1). 65% of these close encounters occur with the Earth in any time period before impact. A small fraction (2.1%, 3.9%, 6.1%, 8.4%, and 11.7% for 10, 20, 30, 40, and 50 years before impact, respectively) of all impactors experience planetary close encounters within 1 $R_H$ of a planet before impact; 90% of these close encounters occur with the Earth.

Of those impactors that undergo planetary close approaches within 1 $R_H$ of a planet, 52%, 66%, 73%, 78%, and 78% for 10, 20, 30, 40, and 50 years prior to impact, respectively, require a deflection $\Delta v$ more than an order of magnitude less than the median required $\Delta v$ at each decade before impact (Table 2). A smaller fraction (7.9%, 10.5%, 13.5%, 14.1%, and 14.6% for 10, 20, 30, 40, and 50 years before impact, respectively) of impactors that experience close encounters within 5 $R_H$ of a planet require a deflection $\Delta v$ more than an order of magnitude less than the median required $\Delta v$.

The smaller distance at closest approach is more heavily dominated by small deflection impulses than the more distant close encounters. This is because the closer distances have a greater effect on the post-encounter asteroid trajectories than more distant encounters. As time before impact increases, opportunities for more distant close approaches increase along with the possibility for multiple close encounters (at any distance) to affect asteroid trajectories. We find that most of the asteroids in our impactor population that experience planetary close approaches have multiple encounters with multiple planets during the 50 years leading up to impact.

Chodas (1999) first introduced the concept of gravitational keyholes, which are small regions of the B-plane such that, if an asteroid passes through one of them, the impact probability of the potential impactor with that planet can be altered, either leading to an impact or a very deep close encounter (Milani et al., 2002; Valsecchi et al., 2003; Chesley, 2006; Farnocchia, 2016). As we find in this study, these planetary close approaches can have a significant effect on the orbital parameters of an asteroid prior to the close encounter, resulting in high sensitivity to their initial conditions and very small $\Delta v$ deflection requirements.

Fig. 7 shows an example of an asteroid with a close planetary approach. This asteroid undergoes a planetary close encounter within 1 $R_H$ of the Earth roughly eight years before impact (or deflection) that significantly decreases its heliocentric distance range. Prior to the close encounter, the asteroid's eccentricity is larger and thus its perihelion

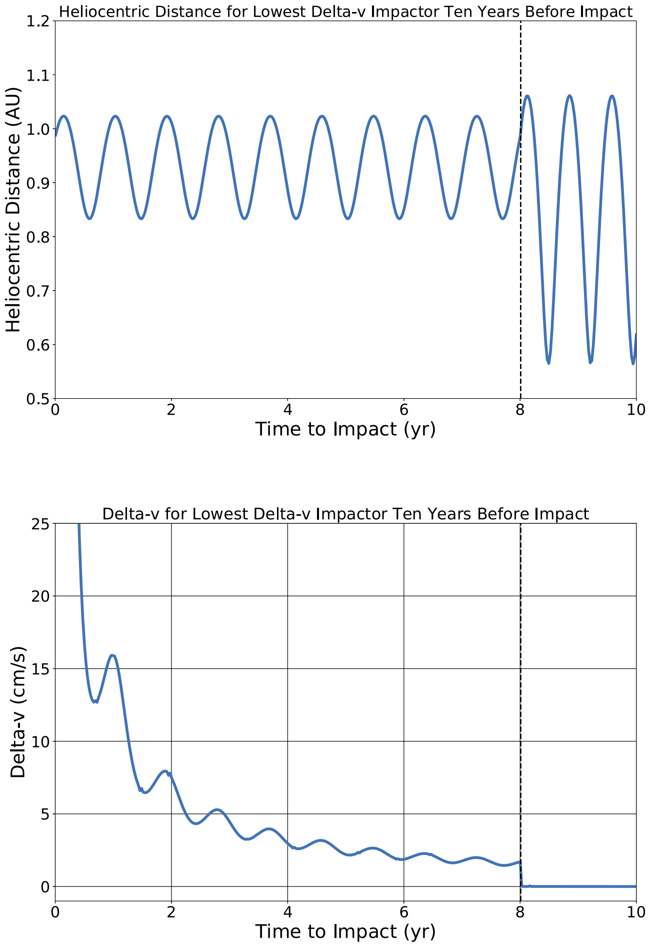

**Fig. 7.** Top: Heliocentric distance as a function of time for the impactor with the lowest computed required $\Delta v$ at 10 years before impact. This asteroid experiences a close encounter with the Earth roughly eight years before impact that significantly decreases the heliocentric range of its orbit. Bottom: Required deflection $\Delta v$ evolution for the asteroid shown above. The close encounter that occurs eight years before impact causes a sudden increase in deflection $\Delta v$.

is smaller, which makes its heliocentric velocity at perihelion higher. Because the deflection $\Delta v$ is computed when this asteroid is very close to perihelion (at $t = 10$ in the figure) and the orbit is moderately eccentric ($e \approx 0.3$), the required deflection $\Delta v$ is therefore low. In fact, this asteroid has the smallest computed $\Delta v$ required of our 10,000 synthetic impactors at 10 years before impact.

The planetary close encounter this asteroid experiences results in an increase in the required $\Delta v$ by a factor of 425 after the close approach occurs. Early detection is thus important for the deflection of this impacting asteroid, as is the need to make decisions regarding mitigation when its impact probability is smaller. This is likely the case for the majority of the asteroids that fall into our lowest-$\Delta v$ bin, all of which experience planetary close encounters within 5 $R_H$.

Some impactors have such sensitive trajectories that the deflection $\Delta v$ required is within numerical noise; when we shifted one of the velocity components of several of these asteroids by ADAM's smallest threshold $\Delta v$ ($1{\times}10^{-7}$ cm/s), many of them no longer hit the Earth. In these cases, the targeter returns no maneuver needed (i.e., $\Delta v < 1{\times}10^{-7}$ cm/s). These impactors compose 0.06%, 0.34%, 1.2%, 3.0%, and 6.3% of the 10,000 impactors for 10, 20, 30, 40, and 50 years prior to impact, respectively. A large fraction of these impactors (100%, 100%, 84%, 72%, and 66% for 10, 20, 30, 40, and 50 years before impact, respectively) also experience close approaches within 1 $R_H$ of a planet (again, roughly 95% of the time with Earth) that alter the asteroid's orbit enough such that the impactor is extremely easy to deflect. Roughly 100% of these impactors undergo close approaches within 5 $R_H$ of a planet, 85% of the time with Earth. These are further examples of the effect that passing through gravitational keyholes (Chodas, 1999; Milani et al., 2002; Valsecchi et al., 2003; Chesley, 2006; Farnocchia, 2016) can have on both the impact probability of an asteroid as well as the deflection $\Delta v$ required to avoid an impact.

The very low required deflection $\Delta v$ make these very low-$\Delta v$ impactors the easiest to deflect (most often when they are discovered early), so one would think these are the type of impactors we want to encounter in a real life impact scenario. However, unfortunately, while these low-$\Delta v$ deflections represent a small fraction of asteroid impact cases, these are also the hardest asteroids to observationally identify as potential threats. This is because the sensitivity of their orbits to initial conditions along with possible planetary gravitational interactions make their future orbits chaotic and much harder to predict than most asteroids. This means more observations will be needed in order to determine the impact probability of these asteroids. Thus early detection will be important both for gaining the increased number of observations required to label these asteroids as impact threats as well as to take advantage of the often significantly easier deflection opportunities before planetary close encounters greatly increase their required $\Delta v$ for deflection. We therefore expect them to dominate the real world deflection decision scenarios we are likely to encounter in the future that will be the most difficult to make. These are thus some of the most important impact deflection scenarios to understand.

### *5.2. Earth impactors that require high-$\Delta v$*

On the other end of the spectrum, we observe a number of impactors that are quite difficult to deflect. The tail of the required deflection $\Delta v$ distribution for all five decades studied before impact extends to 2–3 orders of magnitude larger than the median required $\Delta v$. However, <10% of impactors require more than an order of magnitude more $\Delta v$ than the median at any time before impact. In particular, 1.0%, 2.9%, 4.7%, 6.1%, and 7.8% of impactors for 10, 20, 30, 40, and 50 years prior to impact, respectively, require an order of magnitude higher $\Delta v$ than the median required $\Delta v$ (i.e., $\Delta v \geq 14$ cm/s, 7.6 cm/s, 5.5 cm/s, 4.6 cm/s, and 3.8 cm/s, respectively, for 10, 20, 30, 40, and 50 years prior to impact; see Table 3).

We find that a majority (65%, 66%, 63%, 59%, and 54% for 10, 20, 30, 40, and 50 years before impact, respectively) of these asteroids impact the Earth when their orbits are nearly parallel to that of the Earth (i.e., the heliocentric velocity vector at infinity for the asteroids is within 15° of the Earth's velocity vector; see Table 3). The bottom panel of Fig. 8 shows the distribution of separation angles between the heliocentric velocity vector of the asteroid at infinity and that of the Earth for those impactors with an order of magnitude higher $\Delta v$ than the median at 10 years before impact ($\Delta v \geq 14$ cm/s). 65% of these impactors have velocity vectors separated by <15° from those of the Earth at the time of impact, while 75% have relative separation angles <20°. The top panel of Fig. 8 shows the distribution of angles between the heliocentric velocity vectors of the asteroid at infinity and the Earth for all 10,000 synthetic Earth impactors one day before impact. 41% of all impactors have their velocity vectors separated from that of the Earth by <15° before impact, and 60% have relative separation angles <20°.

Fig. 9 provides the distribution of the required deflection $\Delta v$ when applied 10 years prior to impact as a stacked histogram separated by the angle between the heliocentric velocity vectors of the asteroids at infinity and the Earth one day before impact. For typical Earth impactors that fall within the peak of the $\Delta v$ distribution, impacts occur at a range of angles from 0° – 90°. However, as the deflection $\Delta v$ increases, the angle between the velocity vectors of the two bodies decreases.

The reason these high-$\Delta v$ impactors with nearly parallel orbits to the Earth at the time of impact are harder to deflect than the typical

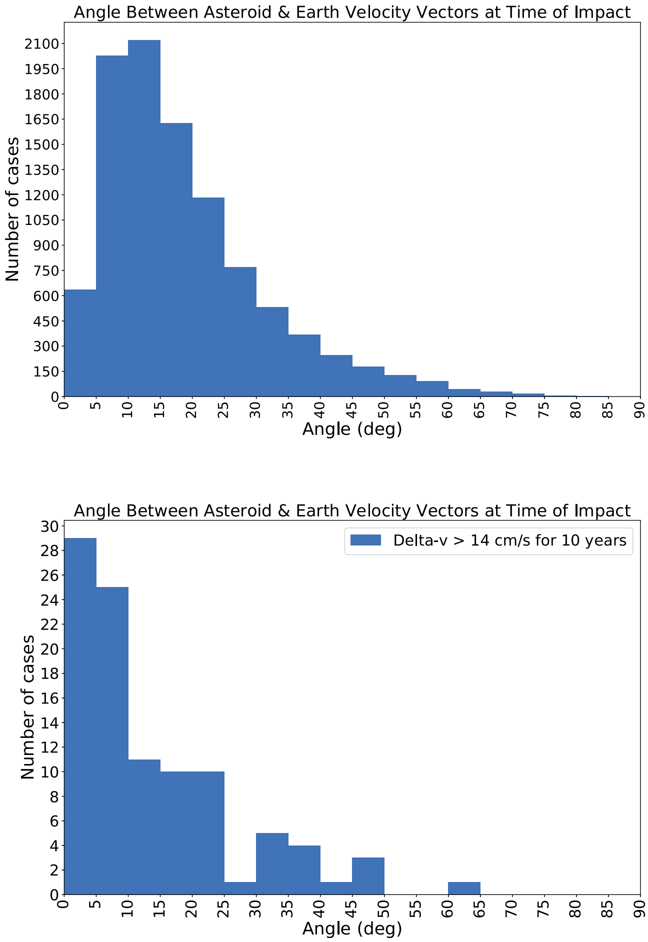

**Fig. 8.** Histograms of the angle between the heliocentric velocity vectors of an asteroid at infinity and Earth one day before impact. Top: Impact angle distribution for all 10,000 synthetic impactors. Bottom: Impact angle distribution for impactors with $\Delta v > 14$ cm/s (an order of magnitude larger than the median $\Delta v$) at 10 years before impact.

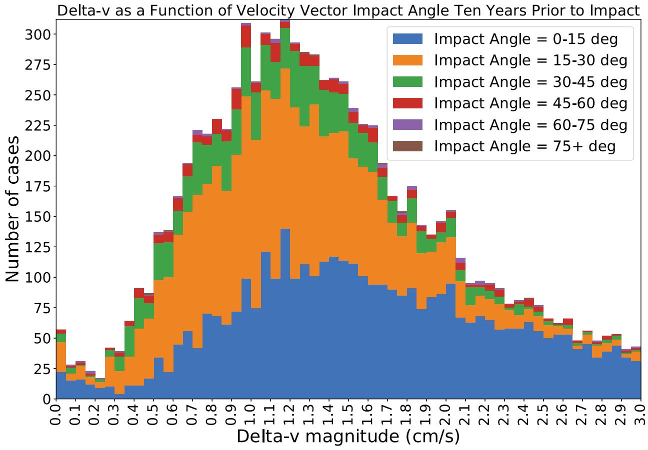

**Fig. 9.** Stacked histogram of the required $\Delta v$ to miss Earth by one Earth radius (i.e., two Earth radii from Earth's center) at 10 years prior to impact separated by the angle between the heliocentric velocity vectors of the asteroid at infinity and Earth one day before impact. The impact angle is split into groups of 15°. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

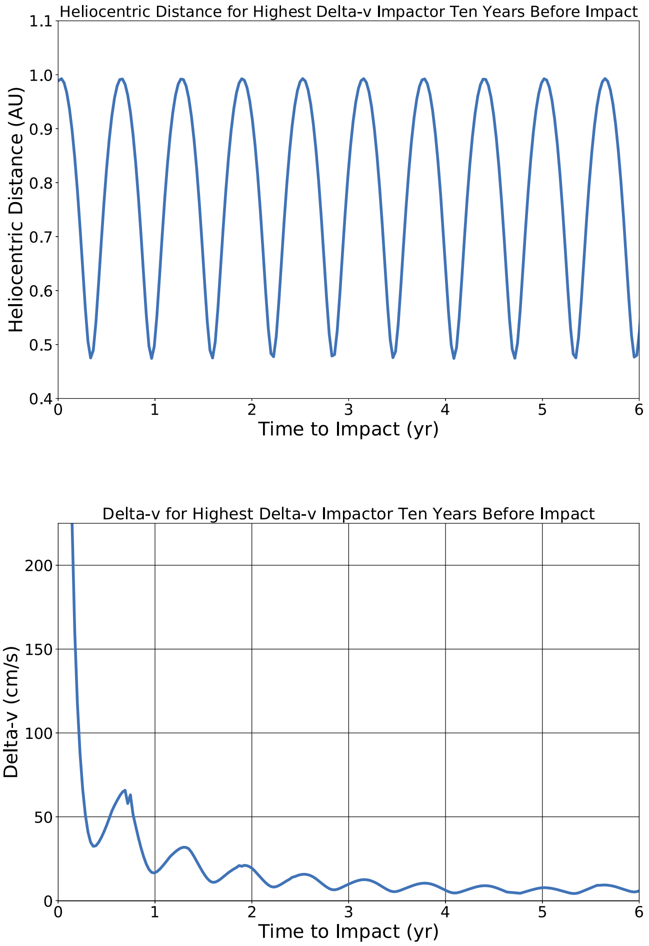

**Fig. 10.** Similar to Fig. 7 but for the impactor with the highest computed required $\Delta v$ at 10 years before impact. The impact between this asteroid and the Earth (or deflection) occurs when the asteroid is at aphelion and the angle between the heliocentric velocity vector of the asteroid at infinity and that of the Earth is very small (nearly 0°), resulting in a very high required deflection impulse.

Earth impactor is because applying a $\Delta v$ deflection maneuver for an Earth impact in the prograde/retrograde direction primarily only has the effect of either delaying or advancing the timing of the impact. Fig. 10 shows the heliocentric distance and required deflection impulse over time for the asteroid found to have the largest required $\Delta v$ at 10 years prior to impact. This asteroid impacts the Earth at aphelion when the angle between the heliocentric velocity vector of the asteroid at infinity and that of the Earth is nearly 0°. This means the required deflection impulse is computed when the two orbits are nearly parallel, resulting in the high $\Delta v$.

We also find that 81%, 68%, 59%, 47%, and 43% of impactors with more than an order of magnitude above the median $\Delta v$ for 10, 20, 30, 40, and 50 years prior to impact, respectively, undergo planetary close approaches within 5 $R_H$ (Table 3). Roughly 2% of high-$\Delta v$ impactors undergo close encounters within 1 $R_H$ of a planet at any decade before impact studied. Of those impactors that undergo planetary close approaches within 5 $R_H$ of a planet, 69%, 61%, 44%, 27%, and 27% for 10, 20, 30, 40, and 50 years prior to impact, respectively, require a deflection $\Delta v$ more than an order of magnitude above the median required $\Delta v$ at each decade before impact (Table 3).

As is true for the low-$\Delta v$ impactors, these planetary close encounters can also have a significant effect on the orbital trajectories of high-$\Delta v$ impactors as well as their required deflection impulses. Contrary to the case of gravitational keyholes, which often make deflection easier at earlier times (Chodas, 1999), some close encounters can make deflection easier at later times. Chodas (2012) coined the term 'Jabbas' to define regions in the target plane that have the undesirable property of weakening prior deflections due to planetary close approaches; passing through a Jabba thus means that at times before the close encounter, the required deflection impulse is greater than that needed post-encounter. As a result, deflection attempts are more effective at times after the encounter rather than before, contrary to the case for keyhole passage. However, even though passage through a Jabba can make deflection easier at later times, it is only in a relative sense. The deflection is still very expensive and difficult due to the fact that Jabbas often focus an asteroid's trajectory toward collision (Milani et al., 2009; Chodas et al., 2008). Those impactors in our study that experience close approaches and require a high $\Delta v$ are likely a result of the Jabba effect and account for a large fraction of the high $\Delta v$ impactors that have angles between their heliocentric velocity vectors at infinity >20° from that of the Earth.

### 5.3. Non-converging cases

One issue we found is that our targeting method failed to converge in a small number of cases. These "failed maneuvers" consisted of 0.46%, 0.70%, 1.11%, 1.51%, and 2.30% of the impactor population when computing the required deflection $\Delta v$ at 10, 20, 30, 40, and 50 years before impact, respectively. Largely these cases are similar to the high-$\Delta v$ cases described in Section 5.2 with perihelia or aphelia at or near the Earth's orbit and heliocentric velocity vectors at infinity just prior to impact nearly parallel to that of the Earth. We find that not only do such cases result in large required deflection impulses as discussed earlier, but the resulting geometry makes the targeting of deflection maneuvers very sensitive to initial conditions, which can cause our targeter to fail to converge.

For many of these impactors, when one component of their initial velocities is very slightly altered by $1\times10^{-7}$ cm/s as above or the initial miss distance used in the targeter is slightly changed, the targeter is then able to converge on a required maneuver without issue. Others have been successfully targeted by manually adjusting the way the algorithm calculates the partial derivatives and controlling the step size between iterations. In addition, using three orthogonal components of $\Delta v$ instead of just the along-track component helped in other cases. In these tests, the difference between the along-track component alone and the three orthogonal components were at most a couple cm/s in the deflection impulse required. Whether the single component or three component method was better appeared to depend on the geometry of the intersecting orbits at the point of impact. Tough geometries, such as impacts occurring at the asteroid's aphelion when the orbits are nearly parallel, often required slightly less deflection impulse when computed using the along-track component alone compared to the three orthogonal components while those with larger angles at the orbital intersection point favored the orthogonal method over the along-track method. Some of these latter cases, and others, may also be solved if the maneuver is moved by several months to a different place in the orbit. Overall, we find a small fraction (<2.5%) of impactors pose particularly tricky orbital geometry at the time of deflection that makes calculating a required deflection $\Delta v$ difficult.

## 6. Conclusions

We are building a precision cloud-based orbit propagation and impact probability computation platform with targeting capabilities called ADAM (Asteroid Decision Analysis and Mapping). The cloud-based implementation of ADAM provides fast orbit propagation of large-scale populations of asteroids, such as is needed for studies like the one presented in this paper on impact deflection requirements as a function of time before impact, due to its ability to parallelize computations on a large number of compute cores. Upon completion of development and testing, ADAM will be made available to the scientific community as open-source software as well as a service.

We find that the median required deflection $\Delta v$ decreases as $\approx t^{-1}$ for increasing time prior to impact in agreement with the order-of-magnitude two-body approximation from Ahrens and Harris (1992). The median required $\Delta v$ when applied 10, 20, 30, 40, and 50 years before impact is found to be 1.4 cm/s, 0.76 cm/s, 0.55 cm/s, 0.46 cm/s, and 0.38 cm/s, respectively. In addition, we find an increasing fraction of impactors that require more than an order of magnitude less $\Delta v$ than the median required $\Delta v$ with increasing time before impact. In particular, 1.2%, 3.3%, 6.4%, 10.1% and 15.0% of impacting asteroids for 10, 20, 30, 40, and 50 years before impact, respectively, require >10 times less velocity impulse than the median $\Delta v$ for each decade before impact studied.

A majority of the lowest deflection $\Delta v$ impactors undergo intervening planetary close approaches that substantially raise their required deflection impulses as a function of time before impact. These asteroids are more easily deflected at times before the close encounters occur, making early discovery of these asteroids important. These asteroids are also very sensitive to their initial conditions, making them easy to deflect, but their trajectories more difficult to determine, resulting in a greater number of observations required to determine whether or not they will impact the Earth.

At the high end of the $\Delta v$ distributions, we find a long tail for all five decades studied prior to impact. These tails stretch out to $\Delta v$ equal to tens of m/s, although we find that <10% of impactors require an order of magnitude higher $\Delta v$ than the median required $\Delta v$ for any time prior to impact. These asteroids often have impacts that occur when the asteroid's heliocentric velocity vector at infinity is nearly parallel to that of the Earth or their deflection maneuvers calculated before an upcoming planetary close encounter that would decrease the required $\Delta v$, making them more difficult to deflect than the typical impactor.

We also find a small number of cases for which the ADAM targeter fails to converge on a deflection maneuver. These cases make up no more than ≈2.5% of impactors for any time before impact. Like the impactors that require high $\Delta v$, these impactors most often have perihelia or aphelia at or near the Earth's orbit and velocity vectors at infinity just prior to impact nearly parallel to that of the Earth. As with the low- and high-$\Delta v$ impactor cases discussed, these impactors are very sensitive to initial conditions, which manifests in sensitivity to the targeting algorithm.

Along with the distributions of the required deflection impulses as a function of time before impact, we discuss the possible atypical impact scenarios we could encounter in the future, such as impactors that are either much more difficult to deflect than the typical impactor or those much easier to deflect but for which impact probabilities are difficult to determine. These atypical impact scenarios should be studied further by the community in order to be more prepared for dealing with any possible real life impact scenario.

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